

Evolution of low-mass metal-free stars including effects of diffusion and external pollution

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ABSTRACT

We investigate the evolution of low-mass metal-free Population III stars. Emphasis is laid upon the question of internal and external sources for CNO-elements, which – if present in sufficient amounts in the hydrogen-burning regions – lead to a strong modification of the stars’ evolutionary behavior. For the production of carbon due to nuclear processes inside the stars, we use an extended nuclear network, demonstrating that hot *pp*-chains do not suffice to produce enough carbon or are less effective than the 3α -process. As an external source of CNO-elements we test the efficiency of pollution by a nearby massive star combined with particle diffusion. For all cases investigated, the additional metals fail to reach nuclear burning regions before deep convection on the Red Giant Branch obliterates the previous evolution. The surface abundance history of the polluted Pop III stars is presented. The possibilities to discriminate between a Pop II and a polluted Pop III field star are also discussed.

Subject headings: stars: evolution – stars: interiors – stars: abundances

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1. Introduction

Little is known about the first generation of stars, which formed out of primordial material with a composition resulting from the Big Bang nucleosynthesis. In particular, the formation process of Pop III stars is not understood and important questions remain unanswered. What was the Pop III initial mass function? Did the low- or high-mass stars form first? Did the first Pop III Supernovae pollute the still fully convective low-mass pre-main sequence stars? On the observational side, no $Z = 0$ star has been observed, a fact which is not surprising given the interaction of potential metal-free stars with the galactic interstellar medium over more than 10 billion years. However, there is an increasing wealth of data concerning the ultra metal-poor halo stars ($[\text{Fe}/\text{H}] \lesssim -2.5$), which are definitely metal-poorer than Pop II stars, whose metallicities cluster at $[\text{Fe}/\text{H}] \gtrsim -2.5$. The metal distribution in the atmospheres of ultra metal-poor halo stars (UMPHS) displays an almost pure SNe II signature with r -process element abundances very similar to the solar ones (Ryan et al. 1996; Sneden et al. 1998). Apparently, the envelopes of these stars contain matter processed in only one generation of massive stars. Indeed, Shigeyama & Tsujimoto (1998) argue that individual UMPHS have metal compositions which can be traced back to *individual* Pop III supernovae of type II. From these results one can conclude that the present UMPHS, which evidently must be the low-mass counterpart of a very early generation of stars, might have formed immediately *after* the first SNe. In this case, they evolved as extremely metal-poor stars, such as investigated by Cassisi & Castellani (1993). Alternatively, they might be *true Pop III* stars whose envelopes have been polluted by SNe ejecta after they already had reached the zero-age main sequence, and therefore had no longer been fully convective. The internal, nuclear evolution in this case is that of Pop III stars, even if the outer parts of the envelope shows the presence of metals. It is the second case – polluted low-mass stars of initial metallicity $Z = 0$ (ignoring the 10^{-10} level of initial BBN ${}^7\text{Li}$ and ${}^7\text{Be}$) – which we are investigating in this paper. Our aim is to provide a theoretical background to assess the evolutionary state of the UMPHS.

There is a significant difference from the structural point of view between extremely metal-poor stars and those of zero metallicity, as has been shown in all existing papers dealing with the evolution of metal-free stars (D’Antona 1982; Guenther & Demarque 1983; Eryurt-Ezer & Kiziloğlu 1985; Fujimoto et al. 1990). The reason is the absence of CNO-elements. While in low-mass Pop II stars during the final stages of the main-sequence (MS) evolution the CNO-cycle is stabilizing the core at moderate temperatures, the sole operation of the pp -chains in Pop III stars leads to significantly hotter cores toward the end of core hydrogen burning. (For more massive Pop III stars, see Ezer & Cameron 1971; El Eid et al. 1983; Arnett 1996. Recall that for non-zero metallicity, the CNO-cycle is not only contributing but dominating the energy generation.) Similarly, the hydrogen burning shell of the post-MS stars shows much higher temperatures as for the Pop II case where the CNO-cycle provides the dominating energy source. This implies that if the star had any source supplying enough CNO, it can convert into an extremely metal-poor star and might evolve – after a possible transition phase – in a more standard fashion. The onset of the CNO-cycle is a rather drastic event leading to excursions in the Hertzsprung–Russell-diagram and

to transient convective regions with mixing episodes. The critical mass fraction has been shown to be $\approx 10^{-10}$ and results from the fact that at $T \approx 10^8$ K about 10^{-8} of the solar CNO-abundance is sufficient to lead to the same energy generation as in a star of solar metallicity.

As it has been demonstrated in the early works on Pop III stars, the temperatures at the end of core hydrogen burning in a $1 M_{\odot}$ star (D’Antona 1982; Fujimoto et al. 1990) are high enough to allow the production of the critical carbon abundance through 3α -reactions. Results at lower masses are inconclusive (D’Antona 1982; Guenther & Demarque 1983). In § 2 we will investigate in detail whether other nuclear chains could be additional sources for carbon production, most notably those that start from pp -chain elements like ${}^7\text{Li}$. This is to ensure that the *in situ* production of carbon is treated correctly in our stellar evolution calculations, because it is crucial for predicting whether and when Pop III stars can convert themselves into extreme Pop II stars. In § 3 we will present such up-to-date calculations of Pop III models, which are partially a repetition of the classical work cited, but are also an extension to a variety of masses. In particular we will discuss for which stellar mass *in situ* production of carbon is possible at all.

As the second source for CNO-elements we investigate that resulting from the assumed pollution by nearby supernovae. While the pollution initially affects only the outermost (possibly convective) envelope, atomic diffusion, which is known to be a significant physical process in the Sun, could transport the added metals to the hydrogen-burning regions. This external source of carbon could in particular affect the hydrogen burning shell in evolved stars. § 4 we will present calculations of Pop III stars evolution under these assumptions and discuss our results in connection with the observations of UMPHS. A section devoted to the conclusions follows.

2. Carbon production by nuclear reactions

The dominant reaction for primary carbon production in stars is the 3α -process. Hydrostatic helium burning is usually taking place at $T \approx 10^8$ K, where enough energy is released to provide stellar luminosities. However, already at lower temperatures some 3α -reactions occur. Since hydrogen burning temperatures in Pop I and II stars are below $8 \cdot 10^7$ K, these two burning phases can safely be assumed to be well separated. The situation is different in Pop III stars, where the central temperature T_c is higher and already before the exhaustion of protons can approach values of $7 \cdot 10^7$ K or above (see § 3 and Fujimoto et al. 1990).

At this temperature and at the typical densities of late main-sequence phase cores (10^5 g cm^{-3}) the lifetime of helium against α -capture is of order 10^{12} yrs. Over the remaining 10^8 years of core hydrogen burning the critical amount of carbon of 10^{-10} (mass fraction) can easily be produced, therefore. For standard Pop II stars this might happen as well, although in smaller amounts, but the additional primary carbon produced is negligible compared to the one already present in the initial mixture.

The 3α -process is not the only way to produce carbon at high temperatures. Mitalas (1985)

investigated “unconventional ^{12}C production in Pop III stars” via α -captures on the light elements ^7Be and ^8B , which are present in equilibrium abundances in the pp-II and pp-III chains. The results of those α -captures would be ^{11}B or ^{11}C , which under subsequent p -captures (note that this limits the ability of these reaction chains to produce carbon to stages before the end of hydrogen burning) would create ^{12}C . Since these reactions dominate the 3α process at $T \lesssim 7.6 \cdot 10^7$ K, they might be able to create carbon in sufficient amounts even before the latter process, or – in stars of lower mass not reaching the critical temperatures for the 3α process – might be the only path to carbon production before the end of core hydrogen burning.

Alternatively, in hot pp-chains, ^8B does not β -decay but reacts via $^8\text{B}(p, \gamma)^9\text{C}$. By further α -capture ^{13}N can be created. Wiescher et al. (1989) have investigated these hot pp-chains in detail with a nuclear reaction network. Besides the classical pp-I, -II, and -III chains they identified – as function of T and ρ – additional chains dominating for hotter conditions. The sequence of reactions proposed by Mitalas (1989) was included (termed rap-II and rap-III). Wiescher et al. (1989) were interested in whether the hot pp-chains could produce CNO-isotopes sufficiently fast such that Pop III supermassive stars can experience a thermonuclear explosion during the fast core collapse. For this specific question they found the rap-processes to be important only for temperatures in excess of 10^8 K (see their Fig. 7).

In the present case, however, the problem is related to the competition between pp-chains and 3α reactions in producing carbon during the final phases of core hydrogen burning, i.e. on nuclear time-scales. In order to answer this question and to determine which reaction chains have to be included in the network part of the stellar evolution code, we used the one-zone nuclear reaction network of Weiss & Truran (1990) and included all potentially relevant reactions⁴. The reaction rates were taken from the most recent update of the reaction rate library of F.-K. Thielemann (Thielemann et al. 1987; Thielemann 1996). The full network is shown in Fig. 1. In the lower right corner standard reactions ((p, γ) , (α, p) , etc.) except those involving neutrons are shown. They are included for all nuclei. In addition, several (but not all) other reactions not fitting into this standard reaction scheme are indicated. As an example, consider the 3α -reaction indicated by a long arrow with solid arrowhead. However, also the $\alpha(\alpha, \gamma)^8\text{B}$ is included as an individual reaction (open arrowhead). An important reaction is $^9\text{C} \rightarrow p + 2\alpha + \beta^+$, which inhibits the artificial buildup of ^9C , from which ^{12}C would become possible. Some arrows indicate several possible reaction paths, e.g. from ^4He to ^7Be we have both an ^3He capture or an (α, n) -exchange included. In the lower left corner we also included reactions involving d , for example $^3\text{He}(d, p)^4\text{He}$ or $d(d, n)^3\text{He}$. Inverse reactions are always taken into account as well. The upper right corner contains nuclides included for the CNO-cycles.

The full network needs as input $T(t)$ and $\rho(t)$, which we took from selected evolutionary calculations. Since these were done with our stellar evolution network including only standard

⁴We will use the term *full network* to denote the one-zone extended nuclear reaction network and call the more limited network used in the stellar evolution code the *stellar evolution network*.

pp-, CNO- and 3α -reactions (but treating all of them simultaneously), the full network might be considered to be inconsistent. However, up to a relative mass fraction of CNO-elements of $X_{\text{CNO}} < 10^{-10}$, the energy production of the star and therefore its temperature evolution will not be affected and all other effects resulting from the production of carbon (e.g. changing the molecular weight) are utterly negligible. In addition, the full network calculations have only exploratory character to identify those reactions which must be included in the stellar evolution code for the proper treatment of carbon production.

Primordial ${}^7\text{Li}$ and ${}^7\text{Be}$ – both present at a level of 10^{-10} – are burnt by α -captures at the beginning of the main-sequence phase or, equivalently, at the very beginning of the full network calculations. Mitalas (1985) already demonstrated that their primordial presence does not lead to carbon production during the pre-main-sequence phase. Carbon itself is produced in standard BBN only at a level of 10^{-15} or lower; in inhomogeneous BBN this can rise under very high n/p -ratios to 10^{-12} at most (Thomas et al. 1993). For the full network we assumed a zero

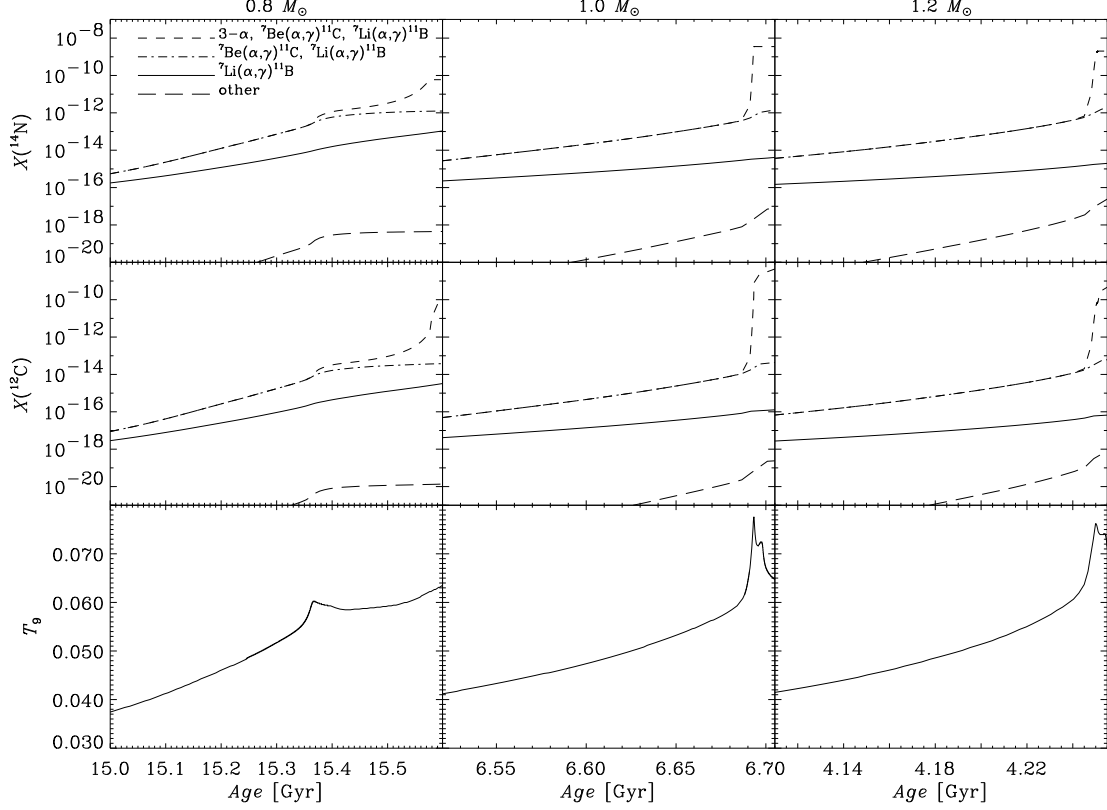


Fig. 2.— Result of the nuclear network calculations for three selected stellar masses. Shown is the ^{14}N (top row) and ^{12}C (middle row) mass fraction as well as the central temperature (bottom row; given as $T_9 \equiv T/10^9$ K) as functions of time

abundance.

The results of the full network calculations are summarized in Fig. 2, which displays the creation of ^{12}C and ^{14}N as a function of time for the last phases of the main-sequence evolution of 3 stars of masses 0.8, 1.0 and $1.2 M_{\odot}$. The bottom panel shows the run of temperature as taken from the stellar evolution calculations including the temperature rise due to the rapid conversion of carbon to nitrogen. The three lines in the upper two panels correspond to the production if all important processes – 3α , $^7\text{Be}(\alpha, \gamma)^{11}\text{C}$ and $^7\text{Li}(\alpha, \gamma)^{11}\text{B}$ – are included (dashed line), if 3α is excluded (dot-dashed), and if only the last one is considered (solid). This latter chain – $^7\text{Li}(\alpha, \gamma)^{11}\text{B}(p, \gamma)^{12}\text{C}$ – turned out to be the next one in the order of decreasing carbon production effectivity. For all three masses, the $^7\text{Be}(\alpha, \gamma)^{11}\text{C}$ chain is the only relevant one for the initially lower temperatures, as is evident from the fact that the dashed and dot-dashed lines lie on top of each other (i.e. the neglect of the 3α -process has no influence), while the solid line ($^7\text{Li}(\alpha, \gamma)^{11}\text{B}$ only) falls below them, even if by less than one order of magnitude. At increasing temperatures towards the end of core hydrogen burning, the 3α -process quickly becomes the most important

source for carbon, which is partially processed to nitrogen, in particular in the more massive stars. The other chains’ contributions decline such that the corresponding lines almost level off. This is due to the exhaustion of protons needed for carbon production in these chains. Note that at earlier times, when enough protons are still available, the conversion of ^{12}C to ^{14}N – the CN-subcycle – is possible such that $^{12}\text{C}/^{14}\text{N}$ is in equilibrium at $\approx 10^{-2}$. With the exhaustion of protons and the additional ^{12}C -source through 3α , carbon finally is exceeding nitrogen in abundance.

While the $^7\text{Be}(\alpha, \gamma)^{11}\text{C}$ is the major path to ^{12}C and ^{14}N for some time, in no case this process is able to create the critical abundance of $\approx 10^{-10}$. Ignoring all processes except 3α therefore introduces only insignificant errors. The relevant carbon production will happen (if it does so at all) through 3α at the end of core hydrogen burning and the carbon missing at that time due to the neglect of the other processes (at a level of 10^{-12} or even less) will quickly be added. Note that the $0.8 M_{\odot}$ star just manages to reach the critical ^{12}C abundance. Due to the lower core temperatures, stars of even lower mass will fail to create enough carbon before the exhaustion of protons such they will not be able to initiate the CNO-cycles. Also, their main-sequence lifetimes are longer than the age of the universe.

From the detailed network computations we have performed we therefore conclude that indeed it is safe to include only the 3α process in the stellar evolution code if one accepts errors of 1% or less in the detailed ^{12}C and ^{14}N production history (which is well contained in the uncertainty of the 3α rate). However, it is also evident that both H- and He-burning reactions must be treated simultaneously in the network.

3. The evolutionary computations: “standard” models

The evolutionary properties of extremely metal-poor stellar structures have been the subject of accurate investigations since the early 1970s, thanks to the pioneering work by Ezer & Cameron (1971). The early investigations have usually been devoted (see the introduction) only to selected evolutionary masses or quite narrow mass ranges; the first complete survey of the evolutionary behavior of extremely metal-poor objects in a quite large range of mass (from low-mass to intermediate and massive stars) has been carried out by Cassisi & Castellani (1993), and was later completed by Cassisi et al. (1996) by extending the numerical computations for low-mass stars to more advanced evolutionary stages, as the central He burning and the double shell H and He burning phases. Following Applegate et al. (1988) the metallicity employed in the computations was $Z = 10^{-10}$, adopted as an upper limit for the cosmological production of heavy elements in inhomogeneous Big Bang nucleosynthesis. In the latter two works a big effort was devoted to investigate all the peculiar evolutionary features related to the paucity of heavy elements in the stellar matter; for this reason a detailed description of the main evolutionary properties of Population III objects will not be repeated here and we refer the interested reader to the quoted papers and reference therein.

In the present work, we decided to adopt a "true" metal free ($Z = 0$) chemical composition. As noted above, the only elements present at a level of 10^{-10} in the Pop III primordial material are ${}^7\text{Li}$ and ${}^7\text{Be}$, which, however, are burnt to helium in the largest parts of the stars already during the pre-main-sequence phase.

The evolutionary models have been computed by adopting an updated version of the FRANEC evolutionary code (Cassisi & Salaris 1997; Salaris & Cassisi 1998). As for the input physics, OPAL radiative opacity tables (Iglesias et al. 1992; Rogers & Iglesias 1992) were adopted for temperatures higher than 10,000 K, while for lower temperatures the molecular opacities provided by Alexander & Ferguson (1994) have been used. Both high and low temperature opacity tables assume a scaled solar heavy elements distribution (Grevesse 1991) when $Z > 0$. The equation of state has been taken from Straniero (1988), supplemented by a Saha EOS at lower temperatures. The outer boundary conditions for the stellar models have been evaluated by assuming the $T(\tau)$ relation provided by Krishna-Swamy (1966). In the superadiabatic region of the stellar envelope a mixing length value of 1.6 pressure scale heights has been adopted.

We emphasize that for some selected evolutionary sequences the numerical computations have also been performed with the Garching evolutionary code (e.g. Schlattl & Weiss 1999) in order to verify the reliability of the present results. In all cases we have obtained good agreement between the two set of "parallel" computations; small differences are due to some differences in the adopted physical inputs. For example, the OPAL EOS (Rogers et al. 1996) is being used in the Garching code.

Following the suggestions by Ezer & Cameron (1971) and by Cassisi & Castellani (1993), local equilibrium values have been adopted for the ${}^3\text{He}$ abundance, independent of the occurrence of convective mixing. If and when the CNO-cycle becomes operative, equilibrium abundances among the various nuclei have been assumed, as derived by solving - for any given temperature and density - the set of equations describing the equilibrium among p -captures and β -decay rates. In both cases, the approach adopted for the chemical equilibrium treatment follows from the evidence that - under the peculiar physical conditions existing in extremely metal-poor stars - the equilibrium timescales are much shorter than the characteristic mixing times (for a detailed discussion on this point we refer to Castellani & Sacchetti 1978)

Taking into account the evidence presented in the previous section that the 3α -reaction is by far the major contributor to ${}^{12}\text{C}$ formation, the evolutionary computations have been carried out by adopting a nuclear network accounting for both H and He-burning, but neglecting any possible unconventional nuclear reactions branch. For all the models an initial helium abundance equal to $Y = 0.23$ - in agreement with the current estimations on the primordial helium abundance - has been adopted.

We have considered stellar masses ranging from $0.8M_{\odot}$ to $1.2M_{\odot}$ (in steps of $0.1M_{\odot}$), and all the evolutionary sequences have been followed from the initial Zero-Age Main Sequence (ZAMS) until the He-burning ignition at the tip of the Red Giant Branch (RGB); in the following, we will

describe the properties of $1M_{\odot}$ stellar models, taken as being representative of this mass range. The evolution of the stars in the considered mass range is quite similar, and the particular choice allows us to make useful comparisons with the results from previous investigations.

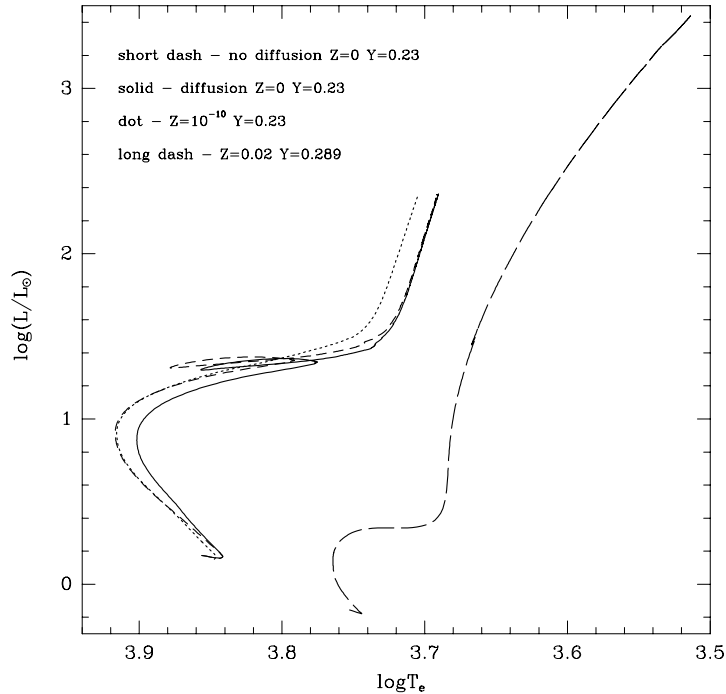


Fig. 3.— The evolutionary paths in the H-R diagram for $1M_{\odot}$ stellar models, computed under different assumptions about the initial chemical composition: canonical metal-free model (short dashed), standard metal-free model accounting for atomic diffusion (solid), canonical $Z = 10^{-10}$ model computed by Cassisi & Castellani (1993) (dotted) and $Z = 0.02$ for comparison (long dashed). All sequences have been followed until the ignition of helium in the degenerate cores

For each mass we have computed two different cases without external pollution: one *without atomic diffusion*, which we denote as ‘canonical’ and one *with atomic diffusion* (the diffusion has been treated according to the formalism by Thoul et al. 1994). The second case we call ‘standard’ following the contemporary definition of the physics for the standard solar model. In addition (see next section), we have computed some evolutionary sequences taking into account both atomic diffusion *and* external He and heavy elements pollution. This case we will call in the following ‘pollution’ or similar. It is worth noticing that this is the first theoretical investigation of Pop III stars accounting in a self-consistent way for the effect of atomic diffusion alone, and of external pollution plus atomic diffusion.

The evolutionary paths of both the $1M_{\odot}$ canonical model and the one computed accounting for helium diffusion (standard), are plotted in Fig. 3. For comparison, we have also plotted the

evolutionary tracks for an extremely metal-poor chemical composition ($Z = 10^{-10}$) as given by Cassisi & Castellani (1993) and for a solar composition, calculated by us with the same program.

Comparing our canonical model with the one by Cassisi & Castellani (1993) it is worth noticing the fine agreement between the two computations, as far as the MS evolution is concerned: the two tracks are perfectly overlapping until the early evolution along the sub-giant branch (SGB). The age of the two models at the Turn Off (TO) is almost the same: $t_H = 6.43$ Gyr for the models computed by Cassisi & Castellani (1993) and $t_H = 6.31$ Gyr for our models; the difference being of the order of a negligible 2%. This result is not surprising since the main difference between the present computations and the ones performed by Cassisi & Castellani (1993) is only the use of different opacity tables. More in detail, Cassisi & Castellani (1993) employed the Cox & Tabor (1976) opacities for $T < 12000$ K, and those by Huebner et al. (1977) at higher temperatures. The evolutionary timescales on the MS and the MS location may be affected by the high-temperature opacities employed in the computations, but the difference between OPAL and Huebner et al. (1977) data is negligible at such low metallicities (see also the discussion in Cassisi & Castellani 1993). On the other hand, the difference in the location of the RGB is due to the higher values of the low-temperature opacities employed in the present calculations, since the low temperature opacities determine the position of the RGB in the Hertzsprung-Russell-diagram (HRD) entirely.

Fig. 3 highlights the evolutionary event, peculiar of true Pop III stars, which occurs during the transition to the RGB: a flash in the H-burning region, which leads to a blue loop HRD. The same result has been obtained also by Fujimoto et al. (1990) by computing a $1M_\odot$ metal-free model. The physical reasons for the occurrence of the flash have been discussed by Fujimoto et al. (1990) and can be summarized as follows. Close to the exhaustion of hydrogen at the center of the star, the core strongly contracts, increasing temperature and density, thereby counterbalancing the effect of the continuous hydrogen depletion on energy production. Due to this increase in both central density and temperature, there is an increase in the efficiency of the triple- α reactions and, in turn, of the abundance of carbon nuclei (cf. Fig. 2). As a consequence, the energy delivered by the CNO cycle strongly increases during this phase. A thermal runaway is produced and a flash occurs which produces the loop of the stellar track in the HRD. In this phase, the star develops a convective core. The ensuing expansion of the inner stellar regions produces a significant decrease in both density and temperature, which reduces the 3α nuclear rate and, as a consequence, also the energy produced by the CN-conversion. When the central abundance of hydrogen drops to zero the convective core disappears.

Even if the general characteristics of this phenomenon are quite similar to the ones obtained by Fujimoto et al. (1990), there are also remarkable differences, which are perhaps due to differences in the physical inputs and in the numerical treatments. In both models, the flash along the SGB occurs when the central abundance of hydrogen is of the order of $X_c \approx 0.0006$. However, the maximum size of the convective core during the thermal runaway is about 50% lower in our computations than in the model computed by Fujimoto et al. (1990) ($\sim 0.11M_\odot$ in comparison with $\sim 0.2M_\odot$). This means that in our model the energy flux produced by the flash

is significantly lower in comparison with previous results. Another significant difference between the two sets of computations is related to the fact that during the runaway the hydrogen-burning rate via the CN cycle is able to exceed the contribution provided by the *pp*-chain in the model produced by Fujimoto et al. (1990). In our computations we find that when the energy delivered by the CNO cycle reaches its maximum it is about a factor 3 lower than the *pp*-chain contribution. Also the maximum helium-burning rate in our models ($L_{\text{He}} \approx 25.7 \cdot 10^{-8} L_{\odot}$) is about two orders of magnitude smaller than the value obtained by Fujimoto et al. (1990).

Due to the general properties of this phenomenon, we suggest that it is not really resembling a He flash like the one occurring at the RGB tip (as suggested by Fujimoto et al. 1990) but that it is more similar to a CNO flash. It is interesting to notice that the chemical abundances of CNO elements are $X_{12\text{C}} = 6.50 \cdot 10^{-12} - X_{14\text{N}} = 2.19 \cdot 10^{-10} - X_{16\text{O}} = 2.77 \cdot 10^{-12}$ at the maximum flash energy production.

Comparing this result with the models provided by Cassisi & Castellani (1993), one notices that the CNO flash along the SGB is missing in the latter computations. A quick comparison between the two models shows that both density and temperature values in the inner core of the stars are significantly lower in the models of Cassisi & Castellani (1993). As we verified with a specific evolutionary computation, this is a result of the fact that Cassisi & Castellani (1993) have assumed a non-zero primordial abundance of heavy elements: even this extremely low metal abundance ($Z = 10^{-10}$) is able to allow the CNO-cycle to operate, such that – at the exhaustion of the central hydrogen abundance – the central temperature remains low enough to avoid the thermal runaway. The initial chemical abundances of CNO elements in the Cassisi & Castellani (1993) model are quite similar to those of our zero-metallicity model *after* the thermal runaway; in this sense, a metal abundance of the order of $Z = 10^{-10}$ seems to be a "critical" metallicity for getting the thermal runaway along the SGB. In any case, one has to bear in mind that the CNO-flash along the SGB is a secondary evolutionary feature which only slightly increases the central hydrogen burning phase duration (of order $10 - 20 \cdot 10^6$ yrs). Therefore, for general purposes as, for instance, population synthesis, the theoretical framework developed by Cassisi & Castellani (1993) and Cassisi et al. (1996) would be applicable also to canonical $Z = 0$ models.

The comparison between the canonical and standard model accounting for atomic diffusion shows the expected changes, i.e. a moderate decrease of the TO luminosity and of the central hydrogen burning lifetime, and a slight shift of the MS toward lower effective temperatures. It is evident that helium diffusion has no effect on the occurrence of the CNO-flash at the end of the central hydrogen burning phase. It is also worth noticing that one has to expect that the efficiency of atomic diffusion during the MS evolution of low mass stars increases when decreasing the stellar metallicity, since the diffusion in the stellar envelopes is larger due to the thinner convective envelopes. Therefore, for fixed diffusion coefficients the effect of atomic diffusion is largest in metal-free stars. This is a quite important point to bear in mind when we will discuss the effect of external pollution on metal-free stars.

As far as the evolution along the RGB is concerned, we did not find any thermal oscillations or instabilities (thermal runaway) in the hydrogen burning shell as discussed by Fujimoto et al. (1990), who emphasized that the occurrence of this phenomenon is strongly related to the choice of the time steps in the numerical computations. However, all numerical experiments we performed have always provided negative results. Overall, there are no significant differences between canonical and standard models along the RGB. This is a well-known fact, since at the base of the RGB the surface convection reaches its maximum extension and mixes back into the convective envelope basically all the chemical elements previously diffused toward the center. The luminosity at the RGB tip (L_{tip}) and the size of the He core at the He-burning ignition (M_{cHe}) shows only minor differences: $M_{\text{cHe}} = 0.497M_{\odot}$ and $\log(L_{\text{tip}}/L_{\odot}) = 2.357$ for the canonical model, and $M_{\text{cHe}} = 0.498M_{\odot}$ and $\log(L_{\text{tip}}/L_{\odot}) = 2.361$ for the standard one. The values of M_{cHe} and $\log(L_{\text{tip}}/L_{\odot})$ appear in fair agreement with the values given by Cassisi & Castellani (1993) in spite of the different assumptions about the primordial metallicity. Some significant differences, however, exist in comparison with the data provided by Fujimoto et al. (1990) and Fujimoto et al. (1995): $\Delta M_{\text{cHe}} \approx -0.015M_{\odot}$ - $\Delta \log(L_{\text{tip}}/L_{\odot}) \approx -0.06$ in the sense that present models have a fainter luminosity and a smaller He core. However, these differences might be understood in terms of changes in the adopted physical scenario.

To close this section, we briefly comment on the helium flash, which terminates the RGB evolution. Fujimoto et al. (1990) and Hollowell et al. (1990) have strongly claimed that during the He flash in a $Z = 0$ model the outer edge of the convective shell formed in the helium zone can extend into H-rich layers, thereby mixing hydrogen back into the helium core. This could produce relevant changes in the surface chemical abundance of the star. In our present computations (but see also the extended survey made by Cassisi & Castellani 1993) we did not find any evidence for this phenomenon, which depends strongly on the competition between convective and nuclear timescale. The former one is not known in the simple mixing-length convection theory, but can only be estimated. Therefore, no firm conclusions about the occurrence of mixing events during the helium flash can be drawn without a more extensive investigation.

4. The non-standard evolutionary models: the effect of external pollution

4.1. Evolutionary properties

The absence of metal-free and the relative paucity of extremely metal-poor stars has been sometimes explained by taking into account the hypothesis of enrichment of the surface metallicity due to accretion of metal-rich material through encounters with interstellar gas clouds. Even if this hypothesis seems to be the most promising one in order to explain the observational evidences, as far as we know it has not been fully investigated until now. The evolutionary consequences and observational implications of such a process have been estimated by Yoshii (1981) and Fujimoto et al. (1995).

In this respect, the work by Yoshii (1981) is quite important as it provides some reasonable estimates of the amount of accreted matter and its heavy elements abundance. The amount of material accreted on a star, due to encounters with gas clouds during its travel through the Galaxy, depends on several parameters as, for instance, the relative velocity between the star and the gas cloud, the cloud parameters and so on. Making some realistic assumptions about the value of these different quantities and the parameters of the stellar orbits, Yoshii (1981) has estimated that the global amount of material accreted on a star with mass of the order of $0.8M_{\odot}$ in a timespan of the order of 10^{10} yr has to be in the range $(10^{-3} - 10^{-2})M_{\odot}$. As far as the metal enrichment is concerned, it strongly depends on the details of the chemical evolution of the Galactic matter. Yoshii (1981) has estimated that after 10^{10} yr, due to external pollution, the surface metallicity of an extremely metal-deficient star should be in the range $0.0006 \leq Z \leq 0.01$ (depending on the stellar orbit properties). The effect on the stellar structures due to atomic diffusion or convective mixing (effectively dilution), however, were taken into account by means of simple considerations based on the known physical properties of standard stellar models for very metal-poor objects. Also in the paper by Fujimoto et al. (1995) the accretion of metals was not treated in a self-consistent way.

For instance, both analyses do not take into account the possible changes in the thermodynamical properties of the envelope due to the change in the opacity of the stellar matter. In addition, if one is interested in investigating in detail the possible changes of the evolutionary behavior of Pop III stars due to accretion of metal-rich matter onto the stellar surface, it is necessary to verify if and when the atomic diffusion is able to bring CNO elements below the edge of the convective envelope and into the H-burning region, which can either be the core or, in later phases, a shell. To test this last point, we have decided to use the most simple accretion model possible. We assume that instantaneous accretion of all the metal-rich matter has happened immediately before the star reached the ZAMS. This means that we simply modify the chemical composition of a certain fraction of the external stellar layers before starting the MS computations. This approach has the benefit to maximize the "efficiency" of the combination of both processes (accretion + diffusion).

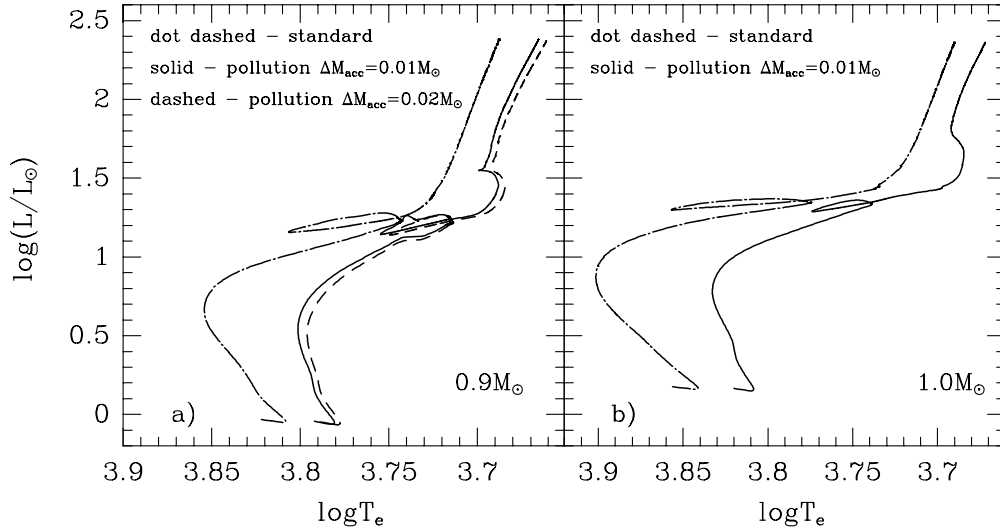


Fig. 4.— (a) The evolutionary paths in the HRD for a $0.9M_{\odot}$ stellar structure, computed under different assumptions about the amount of accreted metal rich matter. (b) as (a) but for the $1M_{\odot}$ model.

In the experiments carried out, we have modified the chemical composition of the outermost $0.01M_{\odot}$ of a model, by adopting as the chemical composition of the “accreted” matter $Z = 0.01$ and $Y = 0.25$. In the following, we are going to describe the resulting evolution of the $0.9M_{\odot}$ and the $1M_{\odot}$ models. In Fig. 4, we have plotted the evolutionary tracks in the HRD and, for comparison, repeated the evolutionary track of the standard metal-free model with atomic diffusion (see § 3).

The main result is that the accreted carbon never reaches the nuclear burning regions. Some interesting features can easily be recognized in the HRD, i.e., the shift toward lower effective temperatures of the models with external pollution. This is due to the increase of the opacities in part of the stellar structure as a consequence of the metals accreted. To be more specific, the lower boundary of the polluted region of the star is located at temperatures higher than 10^6 K, well below the edge of the convective envelope. It is also worth noticing that the occurrence of the CNO-flash along the SGB – an event originated in the deep interior – is not affected at all by the accretion of metals. This is indirect proof that atomic diffusion is not able to bring CNO-elements into the nuclear burning region. In fact, had it been the case, one would have witnessed an increase in the CNO-cycle burning rate, eventually developing into a thermal runaway, during the previous MS evolution. The fact that during the central hydrogen burning phase the evolutionary properties of the polluted stars are not changed⁵ significantly by the diffusion of heavy elements

⁵One can easily notice that the chemical pollution of the outermost layers has the effect of slightly increasing the evolutionary lifetime (see Fig. 5). This occurrence has to be related to the evidence that the polluted models are

– i.e., that atomic diffusion is not efficient enough to bring down the CNO-elements needed to start the CNO-cycle – is also confirmed by the behavior of the various energy sources all along the evolution.

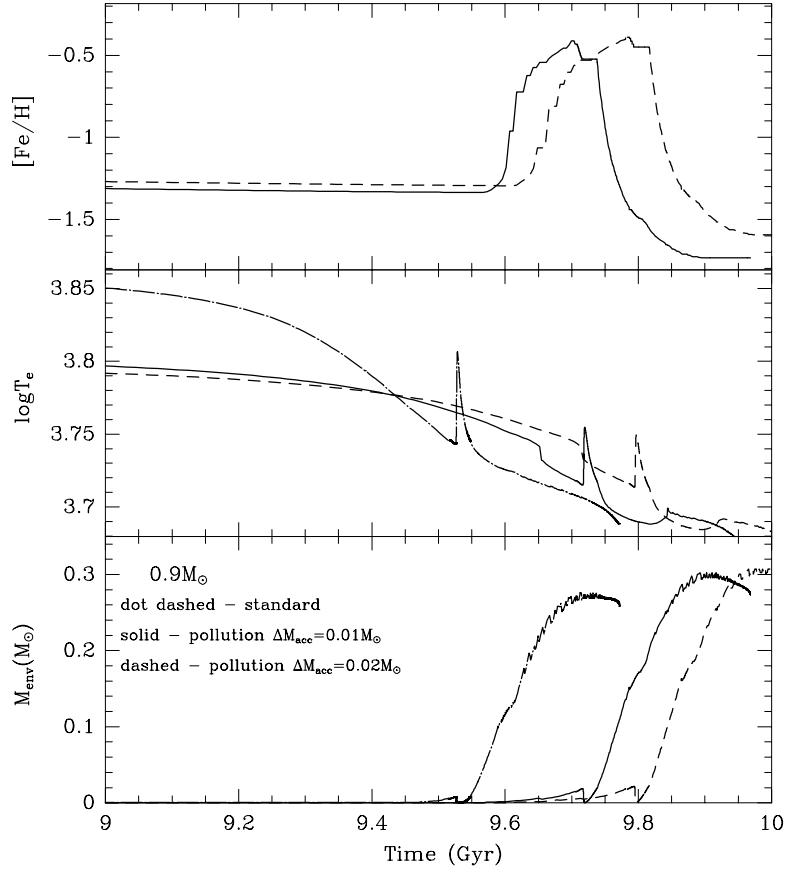


Fig. 5.— Evolution of the surface abundance of iron, the effective temperature, and the size of the convective envelope for a $0.9 M_{\odot}$ model computed taking into account different amounts of accreted metal-rich matter.

During the SGB evolution, before the CNO-flash occurs along the way to the RGB, a thin convective region appears in the envelope (see Fig. 5, lower panel, at $t \approx 9.6$ Gyr in the case of pollution of $0.01 M_{\odot}$). Even if the thickness of this region is very small ($\approx 0.02 M_{\odot}$), basically all the metals diffused during the MS evolution are dredged back to the surface. This is easily noticeable looking at the top panel of the same figure (which displays the run of $[\text{Fe}/\text{H}]$ with time) and also from Fig. 6. The increase in envelope metallicity is accompanied by a decrease in effective temperature. Then the convective envelope goes even deeper, mixing therefore only matter with original zero metallicity; this results in a slight decrease of the surface abundances of

moderately fainter than the standard ones.

the individual heavy-elements at $t \approx 9.7$ Gyr. The maximum abundances reached before this point are never exactly the same as the initial one of the polluting matter, since the diffused metals were already diluted with a metal-free environment.

Then the star experiences the thermal runaway; T_{eff} suddenly increases and the convective envelope disappears, leaving the surface abundances unchanged for a short while. When the runaway stops and the track goes back to the normal evolution toward the RGB, envelope convection sets in another time, this time reaching much deeper ($\approx 0.25 M_{\odot}$). At this point the surface metal abundances decrease even more (more and more metal-free matter is mixed into the convective envelope), while the stellar track moves toward lower effective temperatures because of the progressively larger convective region. When the star is settling on its Hayashi track, the metal abundance in the convective envelope is still decreasing; this produces the "kink" which appears at the base of the RGB in Fig. 4. It is due to the fact that the star is trying to settle on the Hayashi track corresponding to the metallicity of its convective envelope, but the metallicity is still changing due to the deepening of the convective region; therefore the track has to move toward larger T_{eff} , since the RGB-location moves to larger T_{eff} for decreasing surface metallicity. Eventually this process ends when the convective envelope has almost reached its maximum depth, so that a small change in the extension of the convective region does not appreciably change the surface metallicity, and the star starts its standard RGB evolution.

Up to this point our calculations indicate that diffusion is not able to transport the accreted metals into the nuclear burning regions. The last possibility for this occurrence takes place during the RGB evolution, where the hydrogen-burning shell could be able to pass through regions previously reached by the envelope convection at its maximum extension. This effect is present in Pop II stars and is the physical reason for the so-called RGB-bump. Since the envelope metallicity, even if being very diluted with respect to the beginning of the MS phase (by a factor of about 100), is significantly larger than zero, a substantial amount of CNO-nuclei could be ingested into the H-burning shell; this could cause a dramatic change in the efficiency of the H-burning, of the same kind as the one experienced on the SGB. The outcome of our evolutionary computations rules out this possibility, too, even in the very extreme case of our assumptions about the pollution mechanism. In all models we have computed, the distance in mass between the inner point reached by the convective envelope during its maximum penetration and the location of the hydrogen-burning shell at the He-flash, is never less than $\approx 0.1 M_{\odot}$ (we note, in passing, that the absence of the RGB-bump constitutes another difference between Pop III and extreme Pop II evolution). Moreover, we recall that the evolutionary timescale along the RGB is too short for allowing atomic diffusion to bring down CNO-elements from the point of deepest extent of the convective envelope prior to the 3α ignition in the helium core.

We have also checked if a larger amount of metal-rich accreted matter could produce some significant change in the described evolutionary properties of the models. To this end we have doubled the amount of accreted matter ($M_{\text{acc}} = 0.02 M_{\odot}$), which is a factor ≈ 2 larger than the maximum value suggested by Yoshii (1981). One can easily notice from the data plotted in Figs. 4

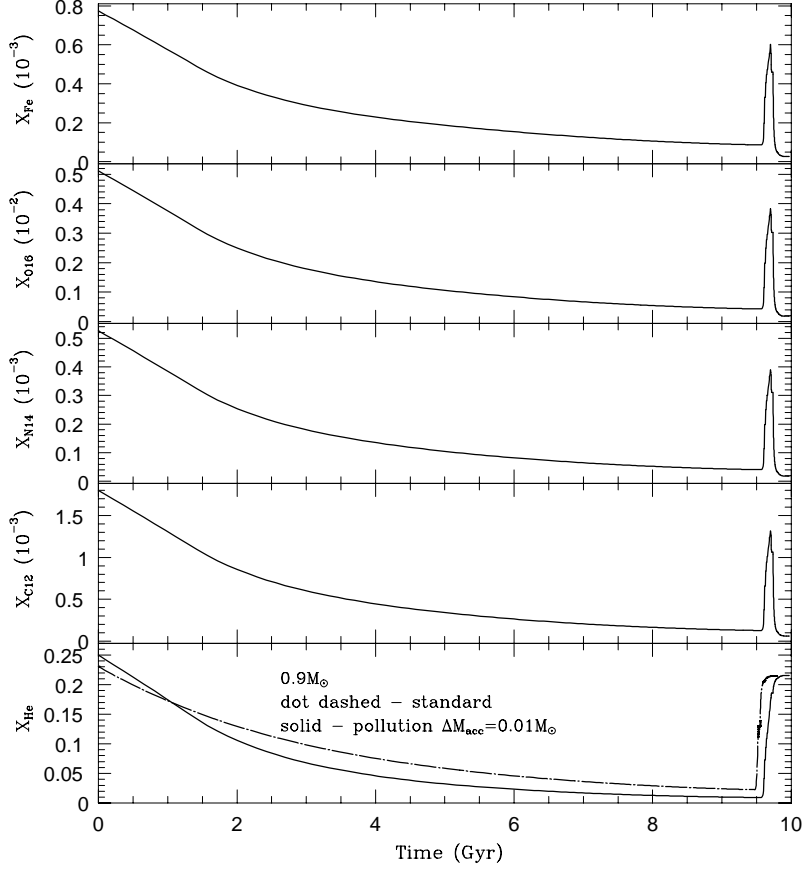


Fig. 6.— Evolution of the mass fractions of CNO-elements, iron and helium, for a $0.9M_{\odot}$ model computed accounting for external pollution. In the bottom panel, the run with time of the surface helium abundance in the model including only diffusion has also been plotted.

and 5 that the effect is very small and that the CNO elements in the accreted matter never reach the H-burning regions.

Taking into account the fact that we have maximized the effects of external pollution plus diffusion, we can safely conclude that the accretion of a significant amount of metal-rich matter on the surface of a metal-free star is not able to change significantly its evolutionary behavior, except for a decrease of effective temperatures (due to the higher metallicity of the external polluted regions) and a small increase in the MS lifetimes. This is mainly due to the fact that atomic diffusion is never efficient enough to bring CNO elements into the inner stellar layers.

4.2. Temporal evolution of the surface metal abundances in polluted metal-deficient stars

Since we have investigated the evolutionary effects of external pollution plus atomic diffusion on metal-free stars by computing self-consistent evolutionary models, we are able to show the run with time of the surface abundances of the most relevant chemical elements. This is a quite important issue since, indeed, it is potentially the only method available for discriminating between polluted Pop III and extreme Pop II stars, when observing isolated field stars.

In Fig. 6 we have plotted the mass fraction of the CNO-elements, helium and iron as a function of time all along the evolution from the ZAMS until He ignition at the tip of the RGB. One can easily notice that as a consequence of atomic diffusion the surface abundances of all these elements are monotonically decreasing during the MS phase. However, as largely discussed in the previous section, when the star approaches the RGB, the convective envelope goes deeper inside the star, dredging up the elements which diffusion has previously carried down in the structure. This occurrence has the effect to produce the sharp increase of the displayed chemical abundances; as soon as the outer convection continues to deepen, it reaches the layers consisting of primordial matter, mixes it with all the outer layers, and the surface chemical abundances sharply decrease again.

The surface abundances evolution of a polluted Pop III star is therefore initially reflecting the pollution process, which we have concentrated into a singular event at the beginning of the star’s MS evolution. Alternatively, continuous accretion or individual pollution events at any time during the MS phase can be envisaged. The pollution effect is modulated by that of diffusion, which will lead to declining metal abundances similar to those as shown in Fig. 6. As soon as the envelope becomes convective – which, in turn, depends on the surface metallicity as well – the previous diffusion history will be obliterated and only the mass ratio between accreted material and the stellar convective envelope will determine the observable abundances along the RGB.

Since the abundance of the single various heavy elements cannot be obtained directly by spectroscopic measurements, we plot in Fig. 7 the expected behavior with time of observable quantities, namely the abundance ratios $[\text{Fe}/\text{H}]$, $[\text{C}/\text{Fe}]$, $[\text{O}/\text{Fe}]$, $[\text{C}/\text{N}]$ and $[\text{O}/\text{N}]$.

From the data plotted in this figure one expects that, when accounting for atomic diffusion and external pollution, Pop III stars should show, during a significant portion of their evolution before reaching the RGB, a slight overabundance of (O/N) and an underabundance of (C/N) , (O/Fe) and (C/Fe) in comparison with the Sun (we recall that a scaled solar abundance has been assumed as the distribution of the polluted matter), and more in general with respect to the same element ratios in the polluting material. However, one has to take into account that the value of these under- and overabundances are quite small (lower than the commonly adopted uncertainty of $\pm 0.10 - 0.15$ dex in spectroscopical measurements) since the diffusion velocities for C, N, O and Fe are quite similar and, moreover, that in our experiments we have maximized the effect of pollution plus diffusion. The element ratio showing the largest difference with respect to its value

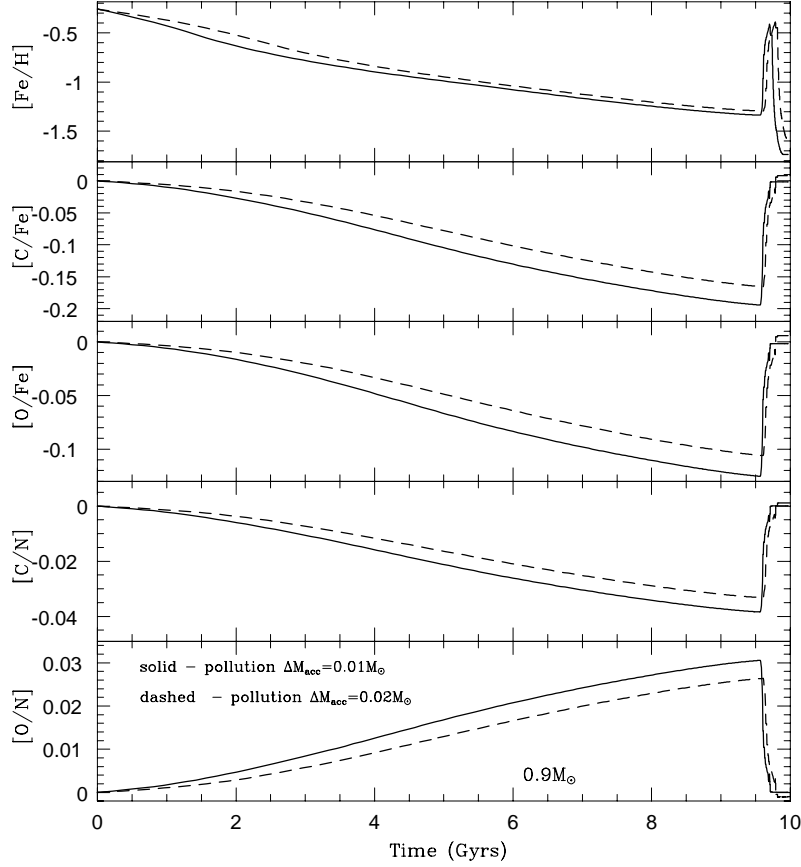


Fig. 7.— Abundance ratios as function of time between CNO elements and iron, and between the CNO elements themselves, for a $0.9M_{\odot}$ model computed by adopting two different assumptions (as labelled) on the amount of polluted matter.

in the polluting matter appears to be $[\text{C}/\text{Fe}]$ (again, of course, only during the MS phase).

Can the measurement of these abundance ratios in MS field UMPHS – in particular $[\text{C}/\text{Fe}]$ – be of some help for discriminating between a very metal poor Pop II MS star and a polluted Pop III one? Probably not very much, for two kind of reasons. The first one is that, as repeatedly stressed before, in our experiments we have maximized the effect of pollution plus diffusion. A steady slow accretion of metals, or an impulsive accretion by discrete amounts (which added together should amount at most to about $0.01M_{\odot}$) would have a smaller effect on the surface abundance ratios, since diffusion has less time to work. Moreover, diffusion should be effective also in Pop II stars and therefore a similar value for, i.e., the $[\text{C}/\text{Fe}]$ ratio, would be observed at the surface of a true Pop II star or of a polluted Pop III one, at least in the simplest hypothesis that the element ratios in the matter from which Pop II stars originated and the matter accreted by Pop III objects are basically the same.

Before concluding this section we want just to recall that, as already emphasized previously, in our computations we have found no evidence for any deep mixing phenomenon at the He ignition, as suggested earlier by Fujimoto et al. (1990) and Hollowell et al. (1990). In the hypothesis that this mechanism is efficient in the cores of Pop III stars, the surface metal abundance (in particular C and N) would be enhanced at the tip of the RGB, in contrast to the effect external pollution, which is washed out during the RGB phase. According to Fujimoto et al. (1995) a nitrogen rich carbon star with $-4.5 \leq [\text{Fe}/\text{H}] \leq -2.0$ must be a Pop III stars. Since the relevance of this issue for the problem of discriminating between metal poor Pop II and polluted Pop III stars, it is our intention to present a detailed investigation of the evolution through the He-flash in a forthcoming paper.

5. Conclusions

The main aim of this work was to investigate the possibility that a low-mass Pop III star could increase its original amount of CNO elements in the core to a level which allows the ignition of the CN cycle, thus modifying its evolutionary behavior from the one characteristic of metal-free object to the one typical of extremely metal-poor stars.

The only two channels allowing – in principle – for this occurrence, are: the production of carbon by means of unconventional nuclear reactions and the accretion of metal-rich matter through encounters with molecular clouds. Both scenarios have been fully explored by means of self-consistent evolutionary computations.

As for the possibility that non-standard nuclear reactions – as the ones suggested by Mitalas (1985) and Wiescher et al. (1989) – could significantly contribute, together with the canonical 3α reactions, to carbon production, it has been found that the most important reaction is ${}^7\text{Be}(\alpha, \gamma){}^{11}\text{C}$, which dominates the carbon synthesis during the early MS phase. However, all non-standard reactions are unable to increase the carbon abundance to the level of 10^{-10} , needed to ignite the CN cycle. Therefore we conclude that, at least in the explored mass range, it is necessary only to account for the 3α reactions in the evolutionary models in order to obtain a reliable estimation of the carbon production. The uncertainty on the final abundance by mass of ${}^{12}\text{C}$ or ${}^{14}\text{N}$, is of the order of a quite negligible 1%. This occurrence can be considered as plain evidence for the reliability of current stellar models for metal-free objects.

The scenario in which a metal-free object accretes metal-rich matter through encounters with molecular clouds, has been investigated by computing evolutionary models accounting for both atomic diffusion and chemical pollution, and by using reasonable estimates of the amount of accreted matter. The numerical computations have clearly shown that, neither atomic diffusion during the MS phase nor outer convection during the RGB evolution, are able to bring CNO-elements into the nuclear burning region; in spite of the remarkable changes of the heavy elements abundance of the outer layers, the evolutionary behavior of a polluted Pop. III star is

always regulated by the original chemical composition.

Since we have investigated only comparably well-known particle-transport effects (convection and diffusion), one could envisage that non-standard effects (for example, mixing induced by rotation) help in transporting metals from the stellar surface to the nuclear processing regions. Indeed, in globular cluster stars there is strong evidence for such an additional mixing mechanism, whose signature is evident by surface anomalies in CNO-elements as well as Na and Mg (see Kraft 1994 for a review). This mixing is simulated in theoretical models (e.g. Denissenkov & Weiss 1996) by an additional diffusion process. The intention and result of these simulations is to transport material from the hydrogen shell to the bottom of the convective envelope. In our case, the direction would be opposite, but the mechanism could be the same. While we cannot exclude such an additional effect completely, there is a strong argument against its occurrence in metal-free stars: there are observational evidences (Suntzeff 1981; Gilroy & Brown 1991) supported by theoretical modelling and considerations (Sweigart & Mengel 1979; Charbonnel 1995) that the process starts only *after* the red giant bump, i.e. when there is no molecular weight gradient (barrier) between the outer shell and the rest of the envelope. As demonstrated, true Pop III stars, however, never reach this phase because helium ignition sets in long before at very low luminosities compared to those of Pop II RGB-tips. This, in turn, is a direct consequence of the hotter cores, the temperature of which is determined by the shell temperatures. As we know, these are higher for shells burning hydrogen via the *pp*-chains. We therefore consider it unlikely that an additional mixing as in globular cluster red giants would appear in Pop III giants.

From the point of view of an ‘external’ observer it is extremely difficult to discriminate between a polluted Pop III field star and a very metal poor Pop II one; in spite of substantial differences in the core physical conditions and energy production mechanisms, the evolution in the HRD is qualitatively quite similar. The effective temperature of the star is basically regulated by the surface metallicity, and the only feature peculiar of the Pop III object is the CNO flash along the SGB, a very fast and unobservable phase. Also the study of the surface abundance ratios does not appear to be of very much help. This means that, if the chemical pollution of the stellar surface is effective, the still surviving Pop. III stars could be all disguised as extremely Pop. II stars with no chance to discriminate between “true” extremely metal-poor stars and polluted Pop. III objects. The only possibility, as mentioned in the previous section, is the occurrence of deep mixing phenomena at the He ignition, which would produce a nitrogen rich metal poor carbon star; this is a subject on which we will present a detailed investigation in a forthcoming paper. Our models also predict a general trend for the surface metal abundance of polluted Pop III stars: it should steadily decrease with evolutionary phase, with the exception of a brief episode during the subgiant phase, when they are higher by a factor of ten (Fig. 5). Given that enough UMPHS are observed and that all of them had been polluted by single events during their earliest evolution (Shigeyama & Tsujimoto 1998), this might be observable.

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